

Potensregneregler

Hvis man ønsker mere udfordring kan de første 4 opgaver springes over.

Opgave 1

Angiv uden brug af hjælpemidler den eksakte værdi af nedenstående kvadratrødder - hvis de eksisterer:

$$a = \sqrt{16}, \quad b = \sqrt{64}, \quad c = \sqrt{-4}, \quad d = -\sqrt{4}, \quad e = \sqrt{\frac{1}{9}}, \quad f = \sqrt{\frac{81}{144}}$$

$$g = \sqrt{-0,25}, \quad h = \sqrt{1000}, \quad i = \sqrt{0,0001}, \quad j = \sqrt{\frac{9}{4}}, \quad k = \frac{\sqrt{36}}{2}$$

Opgave 2

Angiv uden brug af hjælpemidler følgende rødder-hvis de eksisterer:

$$a = \sqrt{9} + \sqrt{16}, \quad b = \sqrt{3^2 + 4^2}, \quad c = \sqrt{3 \cdot 12}, \quad d = \sqrt{13 + 3 \cdot 4},$$

$$e = \sqrt{-4^2 + 2^3}, \quad f = \sqrt{12^2 + 5^2}, \quad g = \sqrt[3]{(-2)^6}, \quad h = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

Opgave 3

Afgør uden hjælpemidler, i hvert af nedenstående tilfælde, om udtrykket er defineret (dvs. har mening). Reducér det i givet fald.

$$a = \sqrt[4]{16}, \quad b = \sqrt[3]{-64}, \quad c = \sqrt{(-4)^2}, \quad d = \sqrt{-4^2}, \quad e = \sqrt[3]{125},$$

$$f = \sqrt[3]{\frac{1}{8}}, \quad g = \sqrt[4]{-81}, \quad h = \sqrt[5]{1/32}, \quad i = \frac{1}{\sqrt[3]{216}}, \quad j = \sqrt[4]{\frac{1}{10000}},$$

$$k = \sqrt{8} \cdot \sqrt{32}, \quad l = \sqrt{45} \cdot \sqrt{5}, \quad m = \sqrt{-3} \cdot \sqrt{-3}, \quad n = \sqrt{(-3)^2}, \quad p = \sqrt{-3^2},$$

$$q = \sqrt{\frac{72}{2}}, \quad r = \frac{\sqrt{243}}{\sqrt{3}}, \quad s = \frac{\sqrt{135}}{\sqrt{15}}, \quad t = \sqrt{\frac{8}{-2}}, \quad u = \sqrt{-0}$$

Opgave 4

Reducér følgende udtryk til eksakte værdier (altså ikke decimalbrøker):

$$a = \frac{4\sqrt{8}}{\sqrt{2}}, \quad b = \frac{5\sqrt{20}}{5}, \quad c = \sqrt{8} \cdot \sqrt{18}, \quad d = \sqrt{6} \cdot \sqrt{54},$$

$$e = \sqrt{125} - \sqrt{80}, \quad f = \frac{\sqrt{27} + \sqrt{75}}{\sqrt{3}}, \quad g = \frac{\sqrt{6} \cdot \sqrt{120}}{\sqrt{5}}, \quad h = \sqrt{125} \cdot \sqrt{5},$$

$$i = \frac{\sqrt{32}}{2\sqrt{2}}, \quad j = (\sqrt{6} - \sqrt{5}) \cdot (\sqrt{6} + \sqrt{5}), \quad k = (2\sqrt{6} - 3\sqrt{24})^2, \quad l = \frac{\sqrt{8} \cdot \sqrt{96}}{\sqrt{3}}$$

Opgave 5

Undersøg ved eksakte regninger, om nedestående ligninger er korrekte:

$$\frac{8}{\sqrt{6} - 2} = 4(\sqrt{6} + 2), \quad \frac{6}{\sqrt{45} - \sqrt{5}} = \frac{3\sqrt{5}}{5},$$

$$\frac{6}{\sqrt{45} - \sqrt{5}} = \frac{3\sqrt{5}}{5}, \quad \frac{8 - \sqrt{2}}{8 + \sqrt{2}} = \frac{33 - 8\sqrt{2}}{31}$$

Opgave 6

Reducér følgende udtryk:

$$\frac{\sqrt{x} + \sqrt{4x} + \sqrt{9x}}{\sqrt{x}},$$

$$2y + (\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y}) - (\sqrt{x} + \sqrt{y})^2,$$

$$\sqrt{(x - y)^2 + 4xy}, \text{ idet } x \text{ og } y \text{ er positive tal.}$$

Opgave 7

Reducér følgende udtryk ved hjælp af potensregningerne:

$$a = 5^2 \cdot 5^3, \quad b = 2^4 \cdot 2^5, \quad c = \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2, \quad d = \frac{3^6}{3^4},$$

$$e = \frac{\left(\frac{2}{3}\right)^7}{\left(\frac{2}{3}\right)^6}, \quad f = \left(\frac{2}{3}\right)^4 \cdot \left(\frac{3}{2}\right)^4, \quad g = 2^6 \cdot 5^6, \quad h = \frac{12^3}{2^3},$$

$$i = \frac{35^4}{5^4}, \quad j = \frac{4^2}{\left(\frac{1}{2}\right)^2}, \quad k = (3^4)^5, \quad l = \left(\left(\frac{2}{7}\right)^3\right)^3$$

Opgave 8

Reducer følgende udtryk ved hjælp af potensregnerreglerne - du skal i hvert enkelt tilfælde gøre rede for, hvilken potensregnerregel, der er anvendt (fra nr. 1 til 5):

$$a = 2^3 \cdot 5^3, \quad b = 7^4 \cdot 7^5, \quad c = \frac{5^{11}}{5^9}, \quad d = \left(\frac{1}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^2,$$

$$e = \frac{\left(\left(\frac{1}{4}\right)\right)^8}{\left(\frac{1}{4}\right)^9}, \quad f = \frac{14^6}{\left(\frac{1}{2}\right)^6}, \quad g = \left(\frac{9}{2}\right)^5 \cdot \left(\frac{2}{9}\right)^5, \quad h = \frac{3^3}{6^3}$$

$$h = \frac{3^3}{6^3}, \quad i = \frac{25^3}{5^3}, \quad j = \frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{4}\right)^3}, \quad k = (7^2)^3, \quad l = \left(\left(\frac{6}{3}\right)^4\right)^6$$

Opgave 9

Reducer følgende udtryk ved hjælp af potensregnerreglerne - du skal i hvert enkelt tilfælde gøre rede for, hvilken potensregnerregel, der er anvendt (fra nr. 1 til 5):

$$a = \frac{4^3 \cdot 5^3}{10^3}, \quad b = \frac{(3^4 \cdot 6^4)}{9^4}, \quad c = \frac{9^{11}}{9^9}, \quad d = 6^4 \cdot \left(\frac{1}{3}\right)^4$$

$$e = \frac{\left(\frac{1}{3}\right)^6}{\left(\frac{1}{6}\right)^6}, \quad f = 5^2 \cdot 4^6 \cdot \left(\frac{1}{2}\right)^6, \quad g = \left(\frac{8}{3}\right)^5 \cdot \left(\frac{3}{16}\right)^5 \cdot 2^6, \quad h = \frac{3^3 \cdot 2^3}{6^3},$$

$$i = \frac{25^3}{5^4 \cdot 5^3}, \quad j = \frac{\left(\frac{1}{2}\right)^3 \cdot 8}{\left(\frac{1}{4}\right)^3}, \quad k = ((3^2)^4)^3, \quad l = (1^4)^6$$

Opgave 10

Omskriv følgende udtryk uden brug af hjælpemidler:

$$a = 2^{-3}, \quad b = 3^{-2}, \quad c = 3^0, \quad d = \left(\frac{4}{3}\right)^0 \cdot \left(\frac{1}{4}\right)^{-2},$$

$$e = \left(\frac{1}{3}\right)^{-1}, \quad f = 1^{-3}, \quad g = 2^{-5} \cdot 2^6, \quad h = \left(\frac{1}{2}\right)^0 \cdot 2^4,$$

$$i = 3^{-4} \cdot 6^4, \quad j = \frac{1}{2^{-5}}, \quad k = (2^3)^0$$

Opgave 11

Omskriv uden brug af hjælpemidler, udtrykkene, så de ikke indeholder negative eksponenter

$$\frac{3^{-2} \cdot 13^{-5}}{7^{-3} \cdot 11^{-6}}, \quad \frac{17^2 \cdot 19^{-3}}{3^{-4} \cdot 7^5},$$

$$\frac{a^{-n} \cdot b^{-m}}{c^{-p} \cdot d^{-q}}, \quad \frac{a^n \cdot b^{-m}}{c^{-p} \cdot d^{-q}}$$

Opgave 12

Udregn, uden brug af hjælpemidler, følgende tal:

$$a = 4^{\frac{1}{2}}, \quad b = 81^{\frac{1}{2}}, \quad c = 27^{\frac{1}{3}}, \quad d = 64^{\frac{1}{2}},$$

$$e = -64^{\frac{1}{2}}, \quad f = ((-5)^2)^{\frac{1}{2}}, \quad g = (-8)^{\frac{1}{3}}, \quad h = 32^{\frac{1}{5}},$$

$$i = 3^{-2}, \quad j = 2^{-3}, \quad k = 4^{-\frac{1}{2}}, \quad l = 7^0$$

Opgave 13

Udregn, uden at brug af hjælpemidler, følgende tal

$$a = 8^{\frac{2}{3}} + 16^{\frac{1}{2}} + 27^{\frac{2}{3}} + 36^{\frac{3}{2}}, \quad b = 2^5 + 36^{\frac{1}{2}} + 16^{\frac{1}{4}} - \left(\frac{1}{2}\right)^2 - 5^0,$$

$$c = \sqrt[3]{(-27)^2} + \sqrt[5]{-243^3}, d = 27^{\frac{2}{3}} - 81^{\frac{1}{4}}$$

Opgave 14

Reducer følgende udtryk:

$$\sqrt[12]{a^3 \cdot b^4} \cdot \sqrt[3]{a^2 \cdot b^3},$$
$$\frac{a^{\frac{1}{3}} \cdot a^{\frac{4}{3}}}{a^{\frac{3}{6}} \cdot a^{\frac{5}{6}}}, \quad \frac{a^{\frac{1}{2}} \cdot a^{-1} \cdot \sqrt{a^3}}{(a^8)^{\frac{1}{2}}}$$

Opgave 15

Reducer følgende udtryk:

$$a = \sqrt{\left(2^{\frac{1}{3}}\right)^{\frac{2}{3}}}, \quad b = \frac{2^{\frac{1}{2}} \cdot 2\sqrt{2^3}}{\sqrt[3]{2}}, \quad c = \frac{\sqrt[3]{3} \cdot 3^2}{\sqrt[3]{3} \cdot 3^5},$$
$$d = \sqrt[7]{a^5 \cdot b^3} \cdot \sqrt[14]{a^{-3} \cdot b}, \quad e = \frac{b^{\frac{1}{4}} \cdot b^{-2}}{b^{-\frac{3}{2}}} \cdot b$$

Facit

Opgave 1

$$a = 4, \quad b = 8, \quad c = \text{kan ikke}, \quad d = -2, \quad e = \frac{1}{3}, \quad f = \frac{9}{12},$$

$$g = \text{kan ikke}, \quad h = 10\sqrt{10}, \quad i = 0.01, \quad j = \frac{3}{2}, \quad k = \frac{6}{2}$$

Opgave 2

$$a = 7, \quad b = 5, \quad c = 6, \quad d = 5,$$

$$e = \text{kan ikke}, \quad f = 13, \quad g = 4, \quad h = 1$$

Opgave 3

$$a = 2, \quad b = -4, \quad c = 4, \quad d = \text{kan ikke}, \quad e = 5,$$

$$f = \frac{1}{2}, \quad g = \text{kan ikke}, \quad h = \frac{1}{2}, \quad i = \frac{1}{6}, \quad j = \frac{1}{10},$$

$$k = 16, \quad l = 15, \quad m = 3, \quad n = 3, \quad p = \text{kan ikke},$$

$$q = 6, \quad r = 9, \quad s = 3, \quad t = \text{kan ikke}, \quad u = \text{kan ikke}$$

Opgave 4

$$a = 8, \quad b = 2 * \sqrt{5}, \quad c = 12, \quad d = 18,$$

$$e = \sqrt{5}, \quad f = 8, \quad g = 12, \quad h = 25,$$

$$i = 2, \quad j = 1, \quad k = 96, \quad l = 16$$

Opgave 5

$$\frac{8}{\sqrt{6}-2} = 4(\sqrt{6} + 2) \text{ true}$$

$$\frac{6}{\sqrt{45}-\sqrt{5}} = \frac{3\sqrt{5}}{5} \text{ true}$$

$$\frac{8-\sqrt{2}}{8+\sqrt{2}} = \frac{33-8\sqrt{2}}{31} \text{ true}$$

Opgave 6

$$\frac{\sqrt{x} + \sqrt{4x} + \sqrt{9x}}{\sqrt{x}} = 6$$

$$2y + (\sqrt{x} - \sqrt{y}) \cdot (\sqrt{x} + \sqrt{y}) - (\sqrt{x} + \sqrt{y})^2 = -2 * \sqrt{x} * \sqrt{y}$$

$$\sqrt{(x-y)^2 + 4xy} = \sqrt{x^2 - 2 * x * y + y^2 + 4 * xy}$$

Opgave 7

$$a = 3125, \quad b = 512, \quad c = \frac{1}{32}, \quad d = 9,$$

$$e = \frac{2}{3}, \quad f = 1, \quad g = 10^6, \quad h = 216,$$

$$i = 2401, \quad j = 64, \quad k = 3^{20}, \quad l = \left(\left(\frac{2}{7}\right)\right)^9$$

Opgave 8

$$a = 2^3 \cdot 5^3 = 10^3, \quad b = 7^4 \cdot 7^5 = 7^{20}, \quad c = \frac{5^{11}}{5^9} = 5^2, \quad d = \left(\frac{1}{3}\right)^3 \cdot \left(\frac{1}{3}\right)^2 = \left(\frac{1}{3}\right)^6,$$

$$e = \frac{\left(\left(\frac{1}{4}\right)\right)^8}{\left(\frac{1}{4}\right)^9} = \frac{1}{\frac{1}{4}}, \quad f = \frac{14^6}{\left(\frac{1}{2}\right)^6} = 28^6, \quad g = \left(\frac{9}{2}\right)^5 \cdot \left(\frac{2}{9}\right)^5 = 1, \quad h = \frac{3^3}{6^3} = \left(\frac{1}{2}\right)^3$$

$$i = \frac{25^3}{5^3} = 5^3, \quad j = \frac{\left(\frac{1}{2}\right)^3}{\left(\frac{1}{4}\right)^3} = 2^3, \quad k = (7^2)^3 = 7^6, \quad l = \left(\left(\frac{6}{3}\right)^4\right)^6 = \left(\frac{6}{3}\right)^{24}$$

Opgave 10

$$a = 2^{-3} = \frac{1}{2^3}, \quad b = 3^{-2} = \frac{1}{3^2}, \quad c = 3^0 = 1, \quad d = \left(\frac{4}{3}\right)^0 \cdot \left(\frac{1}{4}\right)^{-2} = \frac{1}{\left(\frac{1}{4}\right)^2},$$

$$e = \left(\frac{1}{3}\right)^{-1} = \frac{1}{\frac{1}{3}}, \quad f = 1^{-3} = \frac{1}{1^3} = 1, \quad g = 2^{-5} \cdot 2^6 = 2, \quad h = \left(\frac{1}{2}\right)^0 \cdot 2^4 = 2^4,$$

$$i = 3^{-4} \cdot 6^4 = 16, \quad j = \frac{1}{2^{-5}} = 2^5, \quad k = (2^3)^0 = 1$$

Opgave 11

$$\frac{3^{-2} \cdot 13^{-5}}{7^{-3} \cdot 11^{-6}} = \frac{7^3 \cdot 11^6}{3^2 \cdot 13^5}$$

$$\frac{17^2 \cdot 19^{-3}}{3^{-4} \cdot 7^5} = \frac{17^2 \cdot 3^4}{19^3 \cdot 7^5}$$

$$\frac{a^{-n} \cdot b^{-m}}{c^{-p} \cdot d^{-q}} = \frac{c^p \cdot d^q}{a^n \cdot b^m}$$

$$\frac{a^n \cdot b^{-m}}{c^{-p} \cdot d^{-q}} = \frac{a^n \cdot c^p \cdot d^q}{a^n \cdot b^m}$$

Opgave 12

$$\begin{aligned}a &= 4^{\frac{1}{2}} = 2, & b &= 81^{\frac{1}{2}} = 9, & c &= 27^{\frac{1}{3}} = 3, & d &= 64^{\frac{1}{2}} = 8, \\e &= -64^{\frac{1}{2}} = -8, & f &= ((-5)^2)^{\frac{1}{2}} = 5, & g &= (-8)^{\frac{1}{3}} = -2, & h &= 32^{\frac{1}{5}} = 2, \\i &= 3^{-2} = \frac{1}{9}, & j &= 2^{-3} = \frac{1}{8}, & k &= 4^{-\frac{1}{2}} = \frac{1}{2}, & l &= 7^0 = 1\end{aligned}$$

Opgave 13

$$\begin{aligned}a &= 8^{\frac{2}{3}} + 16^{\frac{1}{2}} + 27^{\frac{2}{3}} + 36^{\frac{3}{2}} = 233 \\b &= 2^5 + 36^{\frac{1}{2}} + 16^{\frac{1}{4}} - \left(\frac{1}{2}\right)^2 - 5^0 = \frac{155}{4} \\c &= \sqrt[3]{(-27)^2} + \sqrt[5]{-243} = -18 \\d &= 27^{\frac{2}{3}} - 81^{\frac{1}{4}} = 6\end{aligned}$$

Opgave 14

$$\begin{aligned}\sqrt[12]{a^3 \cdot b^4} \cdot \sqrt[3]{a^2 \cdot b^3} &= a^{\left(\frac{2}{3}\right)} * (a^3)^{\left(\frac{1}{12}\right)} * b * abs(b)^{\left(\frac{1}{3}\right)} \\ \frac{a^{\frac{1}{3}} \cdot a^{\frac{4}{3}}}{a^{\frac{3}{6}} \cdot a^{\frac{5}{6}}} &= a^{\left(\frac{1}{3}\right)} \\ \frac{a^{\frac{1}{2}} \cdot a^{-1} \cdot \sqrt{a^3}}{(a^8)^{\frac{1}{2}}} &= \left(\frac{\sqrt{a^3}}{a^{\left(\frac{9}{2}\right)}}\right)\end{aligned}$$

Opgave 15

$$a = \sqrt{\left(2^{\frac{1}{3}}\right)^{\frac{2}{3}}} = 2^{\left(\frac{1}{9}\right)}$$

$$b = \frac{2^{\frac{1}{2}} \cdot 2\sqrt{2^3}}{\sqrt[3]{2}} = 4 * 2^{\left(\frac{2}{3}\right)}$$

$$c = \frac{\sqrt[3]{3} \cdot 3^2}{\sqrt[3]{3} \cdot 3^5} = \frac{1}{27}$$

$$d = \sqrt[7]{a^5 \cdot b^3} \cdot \sqrt[14]{a^{-3} \cdot b} = \left(\left(\frac{b}{a^3}\right)\right)^{\left(\frac{1}{14}\right)} * a^{\left(\frac{5}{7}\right)} * b^{\left(\frac{3}{7}\right)}$$

$$e = \frac{b^{\frac{1}{4}} \cdot b^{-2}}{b^{-\frac{3}{2}}} \cdot b = b^{\left(\frac{3}{4}\right)}$$